Remarks: - if A is a square matrix of order n X n then ADJ(A-λI) is a Matrix Polynomial in λ of degree n-1

$ADJ(A-\lambda I)=B_0+ B_1 \lambda+ B_2 \lambda^2+ B_3 \lambda^3+ \ldots\ldots\ldots\ldots.. B_{n-1}\lambda^{n-1}$

Where $B_0, B_1, B_2, \ldots\ldots\ldots B_{n-1}$ are square matrices of order **n X n**

Cayley-Hamilton Theorem : - Every Square Matrix satisfies its characteristic equation.

Solution

Let A be a **nXn** matrix

Then characteristic equation of A will be of degree **n**

Let $| A- \lambda I |= a_0+ a_1 \lambda+ a_2 \lambda^2+ a_3 \lambda^3+ \ldots\ldots\ldots\ldots.. a_n\lambda^n=0$

be the characteristic equation of A

To Prove

$a_0I+ a_1A+ a_2A^2+\ldots\ldots\ldots\ldots\ldots\ldots..a_nA_n+=0$

We know tha A(adjA)=|A|I

$\therefore(A-\lambda I) (adj(A-\lambda I))= |A-\lambda I|I$

We know that adj(A-λI) is a matrix polynomial in λ of degree n-1

$\therefore(A-\lambda I)=B_0+ B_1 \lambda + B_2 \lambda^2+\ldots\ldots\ldots\ldots\ldots\ldots B_{n-2} \lambda^{n-2}+ B_{n-1} \lambda^{n-1}$

Where $B_0, B_1, B_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots B_{n-1}$ are square matrix of order nXn

Putting values in equation (A) we get

$(A-\lambda I) (adj(A-\lambda I))= |A-\lambda I|I$

⇨ $(A-\lambda I)(B_0+ B_1\lambda + B_2\lambda^2+\ldots\ldots\ldots\ldots\ldots\ldots B_{n-2} \lambda^{n-2}+ B_{n-1} \lambda^{n-1})$
$= (a_0+ a_1\lambda+ a_2\lambda^2+ a_3\lambda^3+ \ldots\ldots\ldots\ldots.. a_{n-1}\lambda^{n-1} +a_n\lambda^n)I$

⇨ $AB_0+ AB_1\lambda + AB_2\lambda^2+\ldots\ldots\ldots\ldots\ldots\ldots B_{n-2} \lambda^{n-2}+ B_{n-1} \lambda^{n-1}$
$- B_0 \lambda - B_1 \lambda^2- B_2 \lambda^3-\ldots\ldots\ldots\ldots B_{n-2} \lambda^{n-1}- B_{n-1} \lambda^n$
$=a_0I+ a_1\lambda I + a_2\lambda^2 I + a_3\lambda^3 I + \ldots\ldots\ldots\ldots.. a_{n-1}\lambda^{n-1} I +a_n\lambda^n I$

Comparing coefficients of like power of λ

We get constant term	$AB_0= a_0I$
λ^1	$AB_1- B_0= a_1I$
λ^2	$AB_2- B_1= a_2I$
λ^3	$AB_3- B_2= a_3I$
………………………	
………………………	
λ^{n-1}	$AB_{n-1}- B_{n-2}= a_{n-1}I$

λ^n $- B_{n-1} = a_n I$

PreMultiplying these equation Successively by I, A, A^2,…………… A^n and adding

We get

$a_0 I + a_1 A + a_2 A^2 + \ldots\ldots\ldots\ldots\ldots\ldots.. a_n A^n = 0$

Hence every square matrix satisfy its characteristic equation